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Ordinal Welfare Comparisons with Multiple Discrete Indicators: A First Order Dominance Approach and Application to Child Poverty*

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Abstract: We develop an ordinal method for making welfare comparisons between populations with multidimensional discrete well-being indicators observed at the micro level. The approach assumes that, for each well-being indicator, the levels can be ranked from worse to better; however, no assumptions are made about relative importance of any dimension nor about complementarity/substitutability relationships between dimensions. The method is based on the concept of multidimensional first order dominance. We introduce a rapid and reliable algorithm for empirically determining whether one population dominates another on the basis of available binary indicators by drawing upon linear programming theory. These approaches are applied to household survey data from Vietnam and Mozambique with a focus on child poverty comparisons over time and between regions.

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1 Introduction

Appropriate poverty measurement remains an active area of research. Traditional models of social welfare and inequality assume one-dimensional indicators, usually based on monetary variables (e.g., Sen, 1973). Nevertheless, poverty (or welfare) has long been recognized as a multi-dimensional phenomenon. Motivated by the desire to consider more dimensions in analyzing social welfare, poverty and inequality (e.g. UNDP, 1990; Sen, 2006) recent literature has frequently focused on multidimensional measures of poverty. For example, Alkire and Foster (2008) and Roelen and Gassmann (2008) apply a weighting scheme to aggregate across multiple indicators of poverty and well-being. Application of a weighting scheme is very convenient and can be easily justified when a reasonably high degree of consensus exists on the appropriate values for weights. Absent such a consensus, application of methods that require weighting schemes can quickly become problematic as alternative weighting schemes may alter conclusions with respect to the welfare rankings of populations. In these cases, it is useful to consider what can be said concerning the welfare status of two populations without making recourse to a weighting scheme.

In response to the challenge of limiting the imposition of subjective assumptions, other contributions have focused on development of "robust" methods for comparing population welfare, poverty and/or inequality with multidimensional data. These methods allow for valid comparisons across broad classes of underlying social welfare functions. Following the seminal work by Atkinson and Bourguignon (1982, 1987) and Bourguignon (1989), recent contributions include Duclos et al. (2006, 2007), Bourguignon and Chakravarty (2003), Crawford (2005), Gravel et al. (2009), Duclos and Batana (2010), Gravel and Mukhopadhyay (2010), and Muller and Trannoy (2011) among others. Still, these contributions apply conditions that are typically formulated in term of specified signs on the second or higher order cross-derivatives of the underlying social welfare functions.

In this paper, we consider the problem of making welfare comparisons between populations in a situation where only ordinal information is available at the micro level in terms of multidimensional (discrete) well-being indicators. The term “ordinal” here means that, for each well-being indicator, the levels can be ranked from worse to better. However, no assumptions are made about the strength of preference for each dimension, nor about the relative desirability of changes between levels within or between dimensions or the complementarity/substitutability between the dimensions.

To accomplish this, we draw upon a concept known in the literature as multidimensional first order dominance (henceforth, FOD). This concept allows us to make welfare comparisons between two populations on the basis of a series of (binary or multileveled) ordinal welfare indicators. In addition, we introduce a rapid and reliable algorithm for empirically determining whether one population dominates another on the basis of available binary indicators by drawing upon linear programming theory.

The FOD approach obviates the need for the analyst to apply an (arbitrary) weighting scheme across multiple criteria or to impose conditions on the social welfare function, which can be a considerable advantage. However, as with any other “robust” method, this gain comes at some cost. First, the procedure may be unable to determine any difference between two populations. In other words, it can happen that population A does not dominate population B and population B does not dominate population A. Hence, the welfare ranking, based on FOD, is indeterminate. Second, as a pure binary indicator, the FOD procedure provides no sense as to the degree of dominance (or similarity) between two populations. Assume population A dominates population B. Without additional information, one does not know whether population A dominates population B by a considerable degree, such that the conclusion of dominance remains even if “large” declines in the individual welfare indicators of population A occur, or whether the conclusion of dominance rests on a knife's edge such that even a “small” decline in any one welfare indicator for population A would lead to an indeterminate outcome.

We mitigate these costs through the application of a bootstrap approach. In particular, repeated bootstrap samples are drawn from the comparator populations, which are often themselves samples of larger populations. When these repeated bootstrap samples are compared, the final output can be interpreted as an empirical probability that population A dominates population B. These probabilities yield significantly more information than the static application of FOD. For example, we may find that occasionally population A dominates population B and occasionally the inverse occurs but most of the time the results are indeterminate. Or, we might find that A dominates B almost always. Or, we may find that the probability that A dominates B is fairly high while the probability that B dominates A is very low or zero. These cases correspond with the conclusion of rough equality, solid dominance of A over B, and likely dominance of A over B respectively.

Finally, if one is willing to accept the probability that A dominates B as a cardinal measure of welfare, one can then easily derive measures that yield cardinal welfare rankings across multiple populations (e.g., all provinces in a country or all countries in a region). Hence, without imposing weights on the various chosen binary welfare indicators that determine all results, one can cardinally rank populations by welfare status.

These approaches are applied to data from Vietnam and Mozambique with a focus on child poverty. With respect to Vietnam, we find strong evidence of improvement in child welfare between 2000 and 2006. At the same time, significant differences in child welfare between regions persist. Overall, there is little systematic evidence that inequality of child welfare measures is decreasing across regions over the same period. With respect to Mozambique, the evidence for gains in childhood welfare indicators between 2003 and 2008 is much more muted (though there is no evidence for deterioration). Available evidence does point to less pronounced, though still large, differences in child welfare indicators across provinces.

The remainder of this article is laid out as follows. Section 2 provides a technical review of the multidimensional first order dominance methodology. Section 3 introduces our case countries, Vietnam and Mozambique, and presents the binary welfare indicators employed to measure child welfare. Section 4 presents results, and section 5 presents concluding remarks and directions for future research.

2 Multidimensional First Order Dominance

FOD comparisons provide a way of comparing multidimensional well-being without relying on *ad hoc* assumptions about individual well-being or social welfare. FOD can be characterized in several equivalent ways, as reviewed in the following paragraphs.

2.1 Definitions

Much research into the nature of distributional dominance concepts has been conducted, and the theory is by now well developed (see e.g. Marshall and Olkin 1979; Müller and Stoyan 2002; Shaked and Shanthikumar 2007 for general treatments). The traditional criterion for one distribution being unambiguously “better” than another is that of first order dominance, in the stochastic dominance literature also known as the *usual (stochastic) order*.

We start by reviewing the classical theory of one-dimensional first order dominance.[1] For our purpose we can focus on a model with only a finite set of possible outcomes for each individual in the population. Assume, therefore, that the distribution of well-being of some population is described by probability mass function f over a finite set of real-valued outcomes X (i.e. $\sum f(x) = 1$ and $f(x) \geq 0$ for all x in X), and another population is described by the probability mass function g. In this one-dimensional case, conditions (a), (b) and (c) below are equivalent:

(a) g can be obtained from f by a finite sequence of bilateral transfers of density to less desirable outcomes.

(b) Social welfare is at least as high for f than for g for any non-decreasing additively separable social welfare function; i.e. $\sum_{x \in X} f(x)w(x) \geq \sum_{x \in X} g(x)w(x)$ for any non-decreasing real function w.

(c) $F(t) \leq G(t)$ $\forall t$, where $F(\cdot)$ and $G(\cdot)$ are the cumulative distribution functions corresponding to f and g.

Intuitively, we could think of condition (a) as one distribution FOD another if one could hypothetically move from one population distribution to the other by iteratively shifting population mass in the direction from a better outcome to a worse outcome. Thus, whenever we are able to observe FOD between two population distributions, the dominating population is unambiguously "better off" than the other.

This fundamental characterization can be extended to a multidimensional setting (e.g. Lehmann 1955, Strassen 1965, Levhari et al. 1975, Grant 1995). Suppose now that f and g

[1] See also Østerdal (2010) for a discussion.

denote multidimensional probability mass functions over a finite subset X of R^n. Then, f first order dominates g if one of the following three equivalent properties (A)-(C) hold.[2]

(A) g can be obtained from f by a finite number of shifts of density from one outcome to another that is worse.

(B) $\sum_{x \in X} w(x) f(x) \geq \sum_{x \in X} w(x) g(x)$ for every non-decreasing real-valued function w.

(C) $\sum_{x \in Y} g(x) \geq \sum_{x \in Y} f(x)$ for any comprehensive set $Y \subseteq X$.[3]

Again, notice by (A) that instances of FOD occur precisely in the cases where one population unambiguously is better off than another population.

2.2 Checking FOD in practice

For empirical work, it is important to be able to determine in an "efficient" way whether one distribution dominates another. In principle, one can check for FOD by directly checking all the inequalities in (C). This is a simple but generally inefficient method, as the number of inequalities to be checked is very large if you have many dimensions and levels. Algorithms dealing with first order dominance have been invented, though most of them are only built for the one-dimensional case (e.g. Bawa et al., 1979; Fishburn and Lavalle, 1995). Preston (1974) and Hansel and Troallic (1978) assert that an algorithm for finding the maximum flow in a properly defined network can be used to determine dominance. More usefully, for the multivariate discrete case, Mosler and Scarcini (1991) and Dyckerhoff and Mosler (1997) show that tests for first order dominance using definition (A) correspond to a linear program. Hence, first order dominance can be verified using a linear programming package. We

[2] The equivalence between (B) and (C) was proved by Lehmann (1955) and re-discovered in economics by Levhari et al. (1975). The equivalence between (A) and (C) has been obtained as a corollary of Strassen's Theorem (Strassen 1965), cf. e.g. Kamae et al. (1977). Østerdal (2010) provides a constructive and direct proof of the equivalence between the multivariate analogues of (A) and (C).

[3] A set Y is comprehensive if $x \in Y$, $y \in X$ and $y \leq x$ implies $y \in Y$.

operationalize the LP technique in GAMS (GAMS Development Corporation, 2008). In our experience, FOD is rapidly and robustly verified using the CONOPT solver (Drud, 2008). As a mathematical form of the FOD problem as a linear program exists in the literature, we opt to provide our operational version in Appendix A.

2.3 Illustration of FOD with binary indicators

To illustrate the concept, let us consider a hypothetical example of two binary 0-1 variables (dimensions) A and B, i.e. $n = 2$ and $X = \{(0,0), (0,1), (1,0), (1,1)\}$.[4] In every dimension, it is useful to think of the outcome 1 as the good outcome (non-deprived) and 0 as the bad outcome (deprived). Thus, the outcome (0,0) for a person means she is deprived in both dimensions; (0,1) means she is deprived in the first dimension and non-deprived in the second dimension, and so forth.

Let f and g be two probability mass functions on X, defined as indicated in Table 1. (The percentages in bold at the right side and bottom of the table indicate the marginal distributions). Note that the marginal distributions are identical for the two distributions, i.e. in a one-dimensional analysis it will not be possible to distinguish the two distributions.

[Table 1 about here]

By condition (C), we have that f first order dominates g if and only if the following four inequalities (i)-(iv) are jointly satisfied:

(i) $g(0,0) \geq f(0,0)$, (ii) $g(0,0) + g(0,1) \geq f(0,0) + f(0,1)$, (iii) $g(0,0) + g(1,0) \geq f(0,0) + f(1,0)$, and (iv) $g(0,0) + g(1,0) + g(0,1) \geq f(0,0) + f(1,0) + f(0,1)$.

[4] An empirical illustration of the 2×2 case is presented in Sonne-Schmidt et al. (2011).

Here, f does not FOD g, nor does g FOD f, since we have $g(0,0) > f(0,0)$ but $f(0,0) + f(1,0) + f(0,1) > g(0,0) + g(1,0) + g(0,1)$.

As a third example of a distribution, let h be the probability mass function given as specified in Table 2.

[Table 2 about here]

Here, distribution h FOD f. This is immediately verified from checking the four inequalities in (C) listed above. Another way of seeing this (by reference to condition (A)), is to observe that we can obtain f from h by moving some probability mass (10%) from the outcome $(1,1)$ to $(0,0)$.

The FOD criterion differs from the criteria for robust welfare comparisons of the Atkinson-Bourguignon type (see the Introduction for further references). The latter are instances of what is also known as orthant stochastic orderings, cf. e.g. Dyckerhoff and Mosler 1997. Orthant orderings are less restrictive (i.e., make stronger assumptions about the underlying social welfare function) than the FOD criterion.[5] In its primary variant, an orthant ordering can be defined as: f dominates g whenever $\sum_{y \leq x} g(y) \geq \sum_{y \leq x} f(y)$ for any $x \in X$. For the 2x2 case, under orthant orderings, conditions (i)-(iii) (without condition (iv)) are necessary and sufficient dominance criteria. Thus in our example f dominates g. For more levels and dimensions, the number of inequalities that needs to be tested for the two types of criteria differs completely in magnitude.

[5] A possible source of confusion is that in the multidimensional context the term "first order dominance" has been used with different meanings. In particular in the economics literature orthant stochastic orderings of the Atkinson and Bourguignon type for welfare comparisons are often referred to as first order dominance criteria. (Second– and higher order dominance criteria are then derived from further assumptions on the underlying social welfare function.)

3 Case Countries and Welfare Indicators

3.1 Case countries

Vietnam and Mozambique are in focus for the empirical analysis. Arndt et al. (2010) describe a number of similarities between Vietnam and Mozambique. In terms of geography, they are both long relatively thin countries with substantial coastline. In terms of recent history, both have conducted socialist experiments and endured brutal and extended periods of warfare. In addition, both formally adopted a much more market oriented economic approach in the same year, 1986. Since the early 1990s, both Vietnam and Mozambique have been among the fastest growing economies in the world. There are structural similarities as well. In both countries, about 70% of the population is rural. Also, the composition of value added across sectors is surprisingly similar (Arndt et al., 2010). Finally, both Vietnam and Mozambique receive significant external resources. Mozambique has been, since the early 1990s, one of the largest aid recipients in the world on a per capita basis. At the same time, Vietnam has been one of the largest aid recipients in absolute terms. When aid to Vietnam is combined with offshore oil revenues, the per capita value of these resources is roughly similar between the two countries.

There are also important differences. Economic takeoff began in earnest earlier in Vietnam. As a result, Vietnam is richer. Population size differs dramatically with the Vietnamese population being about four times larger than the population of Mozambique. At the same time, land area is smaller in Vietnam. Vietnam is one of the most densely populated countries in the world while population density in Mozambique is relatively sparse. Finally, while both countries are investing heavily in education, Vietnam began its economic takeoff with much higher levels of educational attainment and these differences persist. Other social indicators, such as the infant mortality rate and access to health care services, are generally much better in Vietnam for similar reasons.

3.2 Deprivation Indicators

Our research question is to consider the living standards of Vietnamese and Mozambican children through time and across space using a multidimensional approach. To do this, we choose five main indicators of welfare in the spirit of the severe deprivation notion of the Bristol Indicators.[6] They are derived as follows:

Severe water deprivation. Children who only have access to surface water for drinking or for whom the nearest source of water is not within 15 minutes from their dwelling.

Severe sanitation facilities deprivation. Children who have no access to any kind of improved latrine or toilet.

Severe shelter deprivation. Children living in dwellings with more than five people per room (severe overcrowding) or with no flooring material (e.g. a mud floor).

Severe education deprivation. Children who had never been to school and were not currently attending school.

Severe information deprivation. Children who belong to a household where there is not access to a TV set nor to radio.

For Vietnam, we use the Multiple Indicator Cluster Surveys (MICS) from 2000 and 2006. For Mozambique, we use a Demographic and Health Survey (DHS) from 2003 and a MICS from 2008. Further details on the variables used are available from the authors.

[6] For an alternative set of child poverty indicators for the Vietnam MICS 2006 data, see Roelen et al. (2009, 2010).

4 Results

4.1 Descriptive statistics

For the purposes of the analysis presented here, we focus on children aged 7-17.[7] For this age group, we consider the five indicators of well-being presented above. The percentage of children not deprived in each dimension is presented in Table 3.

[Table 3 about here]

With five binary indicators, the number of possible welfare indicator combinations is 2^5=32. The share of children falling into each combination of deprivation indicators is presented in Table 4. The first row of the table illustrates a combination of severe deprivation in all dimensions. This result has a very small probability in Vietnam. In Mozambique, it is about 7% in 2003 with substantial improvement by 2008. The bottom row of the table illustrates the probability of a child not being deprived in any of the five dimensions. Here, the gain in Vietnam is impressive registering an absolute increase of about 26% percentage points, corresponding to a relative change of 100% between the two waves. Mozambique also registers improvement in the final row (child not deprived in all dimensions) though the improvement is marginal.

[Table 4 about here]

[7] We also conducted the analysis for children aged 0-4 years. Four of the five indicators are the same. The education indicator is not relevant for children aged 0-4 years but a indicator of health deprivation is (vaccinations received). Results for the 0-4 age group are qualitatively similar to the results for the 7-17 age group.

4.2 FOD Comparisons

Tables 5 and 6 illustrate the temporal FOD comparisons for Vietnam and Mozambique respectively. In Vietnam, advance in well-being is registered at the national level, in rural zones, and in two regions using the static approach. The bootstrap confirms that the advances at the national level and in rural zones are robust (see Appendix B for details about the bootstrapping approach). Advance in the Mekong River Delta is also robust while advance in the South East region is somewhat more likely than an indeterminate outcome. Positive (empirical) probability of advance is also registered in urban zones and three additional regions. There is essentially no probability of regression through time in any region.

[Tables 5 and 6 about here]

Mozambique registers fewer gains through time. As in Vietnam, there is essentially no evidence of regression through time. Nevertheless, only one province, Niassa, exhibits gains through time using the static approach. The bootstrap indicates that this gain is only somewhat more likely than an indeterminate outcome. There is positive probability of advance at the national level, in rural zones, and in six of eleven provinces. However, these probabilities tend to be quite small. Zambézia province registers about a one in four chance of advance through time.

Based on these indicators, Vietnam appears to have experienced greater success in advancing the well-being of 7-17 year olds over the period 2000 to 2006 than was registered in Mozambique over the period 2003-2008. These results are broadly consistent with the essentially continuous declines in consumption poverty in Vietnam since 1996 (GSO, 2009) and the stagnation in consumption poverty measures found in recent official figures for Mozambique (DNEAP 2010) for the period 2002/03 to 2008/09.

FOD comparisons are also possible across regions for a given point in time. Tables 7-10 show, for Vietnam, regional comparisons for the cases: static 2000, bootstrap 2000, static 2006, and bootstrap 2006 respectively. Tables 11-14 show analogous results for Mozambique. In each case, the row average and the column average is provided. The row (column) average provides the probability that the region dominates (is dominated by) another region.

[Tables 7-10 about here]

Using these metrics, relatively well-off regions should have relatively large row averages while relatively poor regions should have relatively large column averages. In Vietnam, urban zones, the Red River Delta and the South East are shown to be relatively well off. On the other hand, the rural zones, North East, the North West and the Central Highlands are shown to be relatively poor. Consistent with the temporal analysis, the Mekong River Delta is shown to be relatively poor in 2000 but improves to being neither particularly poor nor particularly well-off in 2006.

For Mozambique, the relatively well off regions are the urban zones and Maputo Province and City. Relatively disfavored provinces include rural zones, Tete, Zambézia, Nampula, and Cabo Delgado. In the temporal analysis, Niassa registers improvement through time. This does not show up in the changes in the static spatial row/column averages through time as Niassa becomes dominated by the urban zone (Niassa is 77% rural) while remaining dominated by Maputo Province and Maputo City. Some progress is evident in the bootstrap where Niassa registers a small gain. Zambézia also exhibits a reasonable chance of temporal gain. This is more evident in the inter-regional comparisons. Zambézia province is dominated by other provinces less frequently and less decidedly in 2008 compared with 2003. Despite these gains, Zambézia is the poorest province in both 2003 and 2008 using the column average as the metric.

[Tables 11-14 about here]

Finally, the results potentially provide some indication of trends in inequality. If all regions were nearly equal, then it is likely that no region would dominate any other (at least it would not do so with a very high probability for the bootstrapping case). The values provided in the matrices shown in Tables 7-14 would then tend to be small and with roughly equivalent values for the row and column averages. The average of the row averages (which equals the average of the column averages) provides a measure of total overall registered probability of dominance in the static and in the bootstrap case. An increase (decrease) in this value over time could be taken as an indication of an increase (decrease) in inequality across regions. By this measure, inequality in Vietnam has remained roughly the same (a small increase is found in both the static and in the bootstrap case). Mozambique, on the other hand, registers a fairly dramatic decline in the probabilities of dominance in both the static and bootstrap cases. At the extremes, Maputo Province and City, while still performing relatively well, are less overwhelmingly dominant and Zambézia, while still performing relatively poorly, is less overwhelmingly dominated. In addition, the reductions in dominance do not occur uniquely at the extremes. Of the 14 regions considered (three aggregates and 11 provinces) and focusing on the bootstrap results, 11 exhibit reduced row averages and 11 (not the same 11) exhibit reduced column averages between 2003 and 2008.

5 Conclusions

The FOD criterion is a demanding test for the dominance of a population distribution relative to another in that only a minimum of highly plausible assumptions on underlying social welfare criteria are made. Despite the generality of the criterion, the empirical analysis illustrates that this criterion delivers useful comparisons of populations. This is particularly true in the context of child poverty where the application of one-dimensional (household) income-based welfare/inequality/poverty measures tend to provide too narrow a view given the importance of other indicators such as access to publicly provided goods and services.

While the theoretical underpinnings of the multidimensional FOD criterion have been known and appreciated in the stochastic dominance literature for around half a century, to our knowledge, an empirical implementation of the multidimensional FOD criterion for comparisons of actual population distributions has never been conducted prior to this study. Our findings provide evidence for broad based advance in the welfare of 7-17 year olds in Vietnam using the five chosen indicator variables. Evidence for advance in Mozambique is much more muted. In neither country is there evidence of regression through time. With respect to inequality, regional differences remain relatively constant in Vietnam. In Mozambique, evidence exists for a reduction in regional disparities between 2003 and 2008 though these remain pronounced.

Future research may take several directions. We shall only mention two:

- In our analysis, we have focused on the Bristol indicators for severe child deprivation (adapted to the context of the case countries and available data). While the welfare comparisons are robust for given indicators, changing the indicators themselves may, of course, change conclusions.[8] Our empirical implementation strategy may be adapted to deal with additional (binary or multileveled) indicators. The number of inequalities to be tested for each pair wise comparison of distributions, however,

[8] Some indicators are binary by nature (like education deprivation/school attendance) while other are binary by construction (like shelter deprivation which in part is based on the number of people per sleeping room) and therefore more than two levels could be introduced along each dimension.

increases dramatically with the addition of further levels or dimensions to the existing indicators and fewer FOD are to be expected. Future research may explore the informativeness of the FOD approach when expanding dimensions and levels.

- In the present paper, we focused on a single age group (children aged 7-17) and welfare comparisons within a single country. With the widespread availability of data from Demographic and Health Surveys (potentially supplemented by MICS), the possibility exists to compare target populations across countries. If children remained in focus, it would be possible to consider the evolution of the living conditions of children and develop indicators of the degree of inequality in important indicators of welfare across a broad array of countries.

6 References

Alkire S, Foster J, 2008. Counting and multidimensional poverty measures. OPHI Working Paper.

Arndt C, Garcia A, Tarp F, and Thurlow J, 2010. Poverty Reduction and Economic Structure: Comparative Path Analysis for Mozambique and Vietnam. UNU-WIDER Working Paper WP2010/122.

Atkinson AB, Bourguignon F, 1982. The comparison of multi-dimensioned distributions of economic status. Review of Economics Studies 49, 183-201.

Atkinson AB, Bourguignon F, 1987. Income distribution and differences in need. In GR Feiwek (ed.), Arrow and the foundations of the theory of economic policy, London: Harvester Wheatsheaf.

Bawa VS, Lindenberg EB, Rafsky LC, 1979. An efficient algorithm to determine stochastic dominance admissible sets, Management Science, 609-622.

Bourguignon F, 1989. Family size and social utility: Income distribution dominance criteria. Journal of Econometrics 42, 67-80.

Bourguignon F, Chakravarty S, 2003. The measurement of multi-dimensional poverty. Journal of Economic Inequality 1, 25-49.

Crawford I, 2005. A nonparametric test of stochastic dominance in multivariate distributions. DP 12/05, Department of Economics, University of Surrey.

CHIP, 2004. Children and poverty — some questions answered. Technical Report, http://www.childhoodpoverty.org/index.php/action=documentfeed/doctype=pdf/id=46/.

Drud A, 2008. CONOPT. ARKI Consulting and Development A/S, Bagsvaerd, Denmark.

Dyckerhoff R, Mosler K, 1997. Orthant orderings of discrete random vectors. Journal of Statistical Planning and Inference, 193-205.

Efron B, 1979. Bootstrap methods: another look at the jackknife. Annals of Statistics 7, 1-26.

Efron B, Tibshirani RJ, 1993. An Introduction to the Bootstrap. New York: Chapman & Hall.

Fishburn PC, Lavalle IH, 1995. Stochastic dominance on unidimensional grids. Mathematics of Operations Research 20, 513-525.

General Statistics Office (GSO), 2009. Result of the Survey on Household Living Standards 2008. Hanoi, Vietnam.

Gordon D, Nandy S, Pantazis C, Pemberton S, and Townsend P, 2003. Child Poverty in the Developing World. Bristol UK: Policy Press.

——, 2003. The Distribution of Child Poverty in the Developing World. Bristol UK: Centre for International Poverty Research.

Grant S, 1995. A strong (Ross) characterization of multivariate risk aversion. Theory and Decision 28, 131-152.

Gravel N, Moyes P, Tarroux B, 2009. Robust international comparisons of distributions of disposable income and regional public goods. Economica 76, 432-461.

Gravel N, Mukhopadhyay A, 2010. Is India better off today than 15 years ago? A robust multidimensional answer. Journal of Economic Inequality 8, 173-195.

Hansel G, Troallic JP, 1978. Measures marginales et théorème de ford-fulkerson. Zeitschrift fur Wahrsheinlichkeitstheorie und verwandte Gebiete 43, 345-251.

Hardy GH, Littlewood JE, 1934. Inequalities. Cambridge UK: Cambridge University Press.

Kamae T, Krengel U, O'Brien GL, 1977. Stochastic inequalities on partially ordered spaces. The Annals of Probability 5, 899-912.

Lehmann EL, 1955. Ordered families of distributions. The Annals of Mathematical Statistics 26, 399-419.

Levhari D, Paroush J, Peleg B, 1975. Efficiency analysis for multivariate distributions. Review of Economic Studies 42, 87-91.

Marshall AW, Olkin I, 1979. Inequalitites: Theory of Majorization and its Applications. Mathematics In Science and Engineering, vol. 143.

Mosler KC, Scarcini MR, 1991. Stochastic Orders and Decision under Risk. Heyward, California: Institute of Mathematical Statistics.

Minujin A, Delamonica E, Davidziuk A, Gonzales E, 2006. The definition of child poverty: discussion of concepts and measurements. Environment and Urbanization 18, 481.

Muller C, Trannoy A, 2011. A dominance approach to the appraisal of the distribution of well-being across countries. Journal of Public Economics, forthcoming.

Müller A, Stoyan D, 2002. Comparison Methods for Stochastic Models and Risks. John Wiley and Sons.

Shaked M, Shanthikumar JG, 2007. Stochastic Orders. Springer.

OHCHR, 2002. Draft Guidelines: A Human Rights Approach to Poverty Reduction Strategies. New York: UN.

Preston C, 1974. A generalization of the FKG inequalities. Communications in Mathematical Physics 36, 233-241.

Quirk JD, Saposnik R, 1962. Admissibility and measurable utility functions. Review of Economic Studies 29, 140-146.

Roelen K, Gassmann F, 2008. Measuring child poverty and wellbeing: A literature review. Maastricht Graduate School of Governance Working Paper Series No. 2008/WP001

Roelen K, Gassmann F, de Neubourg C, 2009. The importance of choice and definition for the measurement of child poverty—the case of Vietnam. Child Indicators Research 2(3), 245-263.

Roelen K, Gassmann F, de Neubourg C, 2010. Child poverty in Vietnam: providing insights using a country-specific and multidimensional model. Social Indicators Research 98, 129-145.

Sonne-Schmidt C, Tarp F, Østerdal LP, 2011. Ordinal comparison of multidimensional inequality: theory and application. (A previous version entitled "Ordinal comparison of multidimensional deprivation: theory and application" is available as Discussion Paper 08-33, Department of Economics, University of Copenhagen).

Sen A, 1973. On Economic Inequality. New York: Oxford University Press.

Sen A, 2006. Conceptualizing and measuring poverty. In Poverty and Inequality, D. Grusky and R. Kanbur (Eds.), Stanford University Press.

Strassen V, 1965. The existence of probability measures with given marginals. Annals of Mathematical Statistics 36, 423-439.

UNICEF, 2009. Defining child poverty, Technical Report. http://www.unicef.org/sowc05/english/povertyissue.html.

UNDP, 1990. Human Development Report. New York: Oxford University Press.

——, The state of the world of the children 2005, in Childwood Under Threat, New York, 2005.

Østerdal LP, 2010. The mass transfer approach to multivariate discrete first order stochastic dominance: Direct proof and implications. Journal of Mathematical Economics 46, 1222-1228.

Appendix A: GAMS code for operationalization of FOD test (with up to 7 binary indicators)

```
SETS

  d      dimensions /10,11, 20,21, 30,31, 40,41 50,51, 60,61, 70,71/
  d1(d)  dimension 1 /10, 11/
  d2(d)  dimension 2 /20, 21/
  d3(d)  dimension 3 /30, 31/
  d4(d)  dimension 4 /40, 41/
  d5(d)  dimension 5 /50, 51/
  d6(d)  dimension 6 /60, 61/
  d7(d)  dimension 7 /70, 71/

  off(d1,d2,d3,d4,d5,d6,d7) dimensions not in operation
;

alias(d1,dp1), (d2,dp2), (d3,dp3), (d4,dp4),
     (d5,dp5), (d6,dp6), (d7,dp7);

PARAMETERS
  a(d1,d2,d3,d4,d5,d6,d7)         Matrix that might be revealed superior
  b(d1,d2,d3,d4,d5,d6,d7)         Matrix to test against
;

VARIABLES

   Z degenerate objective
   X(dp1,dp2,dp3,dp4,dp5,dp6,dp7,d1,d2,d3,d4,d5,d6,d7) transport of probability around the lattice
 ;

EQUATIONS
   obj objective
   FOD(d1,d2,d3,d4,d5,d6,d7) equation to check for first order dominance of A over B
;

  obj .. Z=e= 1;

  FOD(d1,d2,d3,d4,d5,d6,d7)$(NOT off(d1,d2,d3,d4,d5,d6,d7)) ..
       a(d1,d2,d3,d4,d5,d6,d7) + sum((dp1,dp2,dp3,dp4,dp5,dp6,dp7)$(
             ord(d1) le ord(dp1) AND
             ord(d2) le ord(dp2) AND
             ord(d3) le ord(dp3) AND
```

```
ord(d4) le ord(dp4) AND
ord(d5) le ord(dp5) AND
ord(d6) le ord(dp6) AND
ord(d7) le ord(dp7)),
X(dp1,dp2,dp3,dp4,dp5,dp6,dp7,d1,d2,d3,d4,d5,d6,d7))

- sum((dp1,dp2,dp3,dp4,dp5,dp6,dp7)$(
ord(dp1) le ord(d1) AND
ord(dp2) le ord(d2) AND
ord(dp3) le ord(d3) AND
ord(dp4) le ord(d4) AND
ord(dp5) le ord(d5) AND
ord(dp6) le ord(d6) AND
ord(dp7) le ord(d7)),
X(d1,d2,d3,d4,d5,d6,d7,dp1,dp2,dp3,dp4,dp5,dp6,dp7))

=e=b(d1,d2,d3,d4,d5,d6,d7);
```

Appendix B: The Bootstrapping

Bootstrapping is a general means of generating consistent estimates of an estimator's sampling distribution when an analytical solution cannot be derived or requires unreasonable assumptions (Efron, 1979; Efron and Tibshirani, 1993). It is based on repeated (J times) samples, drawn with replacement, of size K from the original sample data, of size N, where $K \leq N$. As the original sample size, N, increases, the bootstrap approach converges to Monte Carlo for fixed K. The primary assumption behind the bootstrap is that the distribution of the observed sample is a good approximation of the distribution of the population.

In our application, the bootstrap samples are drawn in a manner that mimics the stratified cluster sample design of the household budget surveys. That is, within each stratum, K clusters are randomly drawn, with replacement, where K is also the number of primary sampling units in the stratum (i.e., K=N). When a cluster is drawn, all of the households in that cluster are drawn. Because the bootstrap sampling is done with replacement, each cluster (and household) may appear one or more times in a given bootstrap sample, or not at all. The FOD analysis using the linear programming techniques discussed in the previous section is conducted for each bootstrap sample. The process is repeated J=1,000 times. The share of times where temporal and/or spatial dominance is discovered over the 1,000 bootstrap replications is then calculated for each result.

Table 1

The distribution for f.

f			Dimension B		Total
			0 (Deprived)	1 (Not deprived)	
Dimension A	0	(Deprived)	25%	25%	50%
	1	(Not deprived)	25%	25%	50%
Total			**50%**	**50%**	**100%**

The distribution for g.

g			Dimension B		Total
			0 (Deprived)	1 (Not deprived)	
Dimension A	0	(Deprived)	35%	15%	50%
	1	(Not deprived)	15%	35%	50%
Total			**50%**	**50%**	**100%**

Table 2

The distribution for h.

h			**Dimension B**		**Total**
			0 (Deprived)	1 (Not deprived)	
Dimension A	0	(Deprived)	15%	25%	40%
	1	(Not deprived)	25%	35%	60%
Total			**40%**	**60%**	**100%**

Table 3

Children not deprived by welfare indicator, 7-17 years old (%).

	Vietnam		Mozambique	
	2000	2006	2003	2008
Water	75.7	87.8	37.6	33.3
Sanitation	37.1	70.9	52.7	60.0
Shelter	60.4	78.4	30.3	46.0
Education	96.0	98.2	76.0	88.4
Information	76.9	87.1	61.8	63.5

Source: Own calculations based on MICS 2 (2000) and 3 (2006) for Vietnam, and DHS 2003 and MICS 2008 for Mozambique.

Table 4

Children by combination of welfare indicators, 7-17 years (%).

Welfare indicator combination					Vietnam			Mozambique		
Water	Sanita.	Shelter	Educa.	Inform.	2000	2006	Change	2003	2008	Change
0	0	0	0	0	1.33	0.22	-1.11	6.92	2.09	-4.83
0	0	0	0	1	1.13	0.08	-1.05	5.88	2.03	-3.86
0	0	0	1	0	5.44	1.24	-4.20	10.80	9.07	-1.73
0	0	0	1	1	8.30	1.68	-6.63	11.17	10.24	-0.94
0	0	1	0	0	0.13	0.02	-0.11	0.50	0.75	0.26
0	0	1	0	1	0.18	0.05	-0.12	0.61	1.03	0.42
0	0	1	1	0	1.97	0.85	-1.12	0.80	2.93	2.13
0	0	1	1	1	4.59	1.79	-2.80	1.33	5.61	4.28
0	1	0	0	0	0.00	0.03	0.03	2.60	1.27	-1.33
0	1	0	0	1	0.00	0.06	0.06	2.41	1.02	-1.39
0	1	0	1	0	0.07	0.40	0.33	5.19	6.83	1.65
0	1	0	1	1	0.22	1.26	1.04	8.43	8.96	0.53
0	1	1	0	0	0.00	0.00	0.00	0.14	0.24	0.10
0	1	1	1	0	0.00	0.05	0.05	0.30	0.55	0.24
0	1	1	0	0	0.12	0.51	0.39	1.35	3.36	2.01
0	1	1	1	1	0.85	3.98	3.13	3.99	10.74	6.75
1	0	0	0	0	0.35	0.35	0.00	0.88	0.76	-0.12
1	0	0	0	1	0.41	0.19	-0.22	0.98	0.25	-0.73
1	0	0	1	0	5.34	3.49	-1.85	2.05	1.72	-0.33
1	0	0	1	1	12.58	4.89	-7.69	2.44	1.57	-0.87
1	0	1	0	0	0.08	0.05	-0.03	0.06	0.15	0.09
1	0	1	0	1	0.23	0.23	-0.01	0.17	0.14	-0.03
1	0	1	1	0	4.19	2.25	-1.94	0.47	0.67	0.20
1	0	1	1	1	16.69	11.72	-4.97	2.19	0.94	-1.25
1	1	0	0	0	0.02	0.01	-0.01	0.77	0.28	-0.49
1	1	0	0	1	0.03	0.13	0.10	0.95	0.39	-0.56
1	1	0	1	0	1.04	1.08	0.03	2.89	3.08	0.19
1	1	0	1	1	3.39	6.51	3.11	5.34	4.39	-0.96
1	1	1	0	0	0.01	0.03	0.02	0.13	0.18	0.05
1	1	1	0	1	0.15	0.29	0.14	0.66	0.41	-0.25
1	1	1	1	0	3.00	2.36	-0.64	2.68	3.13	0.45
1	1	1	1	1	28.16	54.21	26.05	14.88	15.18	0.31
Total					100.00	100.00	0.00	100.00	100.00	0.00

Note: In the first five columns, a "0" means that the child is deprived and an "1" means that the child is not deprived with respect to a given of the five presented welfare indicators.

Source: Same as for Table 3.

Table 5

Temporal FOD comparisons for Vietnam (Probabilities).

	Static case	Bootstrap			
		2006 FOD		2000 FOD	
		2000	Undecided	2006	Total
National	1	1.00	0.00		1
Rural	1	1.00	0.00		1
Urban		0.30	0.70		1
Red River Delta			1		1
North East		0.14	0.86	0.00	1
North West		0.04	0.96	0.00	1
North Central Coast			1		1
South Central Coast			1		1
Central Highlands		0.30	0.70		1
South East	1	0.54	0.46		1
Mekong River Delta	1	0.98	0.02		1

Note: A "1" in the static case indicates that the region's last year welfare level FOD the first year welfare level, while an empty cell indicates no domination. In the bootstrap case a “1” indicate that all 1,000 bootstrap replications resulted in the mentioned domination, while a “1.00” indicate that there were between 995 and 999 dominations, an empty cell indicate that there were no dominations and finally a “0.00” indicate that there were between 1-4 dominations out of a total of 1,000 bootstrap replications.

Source: Same as for Table 3.

Table 6

Temporal FOD comparisons for Mozambique, (Probabilities).

	Static case	Bootstrap			
		2008 FOD		2008 FOD	
		2003	Undecided	2003	Total
National		0.01	0.99		1
Rural		0.08	0.92		1
Urban		0.00	1.00		1
Niassa	1	0.53	0.47		1
Cabo Delgado		0.01	0.99		1
Nampula		0.01	0.99		1
Zambezia		0.24	0.76		1
Tete			1		1
Manica			1		1
Sofala		0.01	1.00		1
Inhambane			1		1
Gaza		0.01	0.99		1
Maputo Province		0.00	1.00		1
Maputo City			1		1

Note: Same as for Table 5.

Source: Same as for Table 3.

Table 7

Spatial FOD comparisons for Vietnam, 2000.

	National	Rural	Urban	RRD	NE	NW	NCC	SCC	CH	SE	MRD	Avg.
National		1			1	1					1	0.40
Rural						1						0.10
Urban	1	1			1	1			1	1	1	0.70
Red River Delta	1	1			1	1	1		1		1	0.70
North East						1						0.10
North West												0.00
North Central Coast									1			0.10
South Central Coast		1				1					1	0.30
Central Highlands												0.00
South East	1	1			1	1			1		1	0.60
Mekong River Delta												0.00
Average	0.30	0.50	0.00	0.00	0.40	0.70	0.10	0.00	0.40	0.10	0.50	0.30

Note: Same as for Table 5.

Averages: The row averages are the fraction of times the row region dominates other regions. The column averages indicates the fraction of times the column region is dominated by other regions.

Source: Same as for Table 3.

Table 8

Bootstrap spatial FOD comparisons for Vietnam, 2000 (Probabilities).

	National	Rural	Urban	RRD	NE	NW	NCC	SCC	CH	SE	MRD	Avg.
National		1			0.47	0.96			0		0.66	0.31
Rural					0.06	0.78					0.26	0.11
Urban	1	1			0.98	1	0.09	0.23	0.91	0.47	1	0.67
Red River Delta	0.99	1			0.94	1	0.39	0.4	0.99	0.02	0.96	0.67
North East	0.01	0.04				0.67	0		0.01		0.15	0.09
North West											0.00	0.00
North Central Coast	0.01	0.07			0.08	0.28		0.01	0.31		0.09	0.08
South Central Coast	0.12	0.52			0.31	0.93			0.18		0.72	0.28
Central Highlands	0.00	0.01			0.02	0.22					0.10	0.03
South East	0.92	0.98			0.88	1			0.62		0.98	0.54
Mekong River Delta						0.04						0.00
Average	0.31	0.46	0.00	0.00	0.37	0.69	0.05	0.06	0.30	0.05	0.49	0.28

Note: Same as for Table 5.

Totals: See explanation in Table 7.

Source: Same as for Table 3.

Table 9

Spatial FOD comparisons for Vietnam. 2006.

	National	Rural	Urban	RRD	NE	NW	NCC	SCC	CH	SE	MRD	Avg.
National		1			1	1						0.30
Rural						1						0.10
Urban	1	1			1	1	1	1	1		1	0.80
Red River Delta	1	1			1	1	1	1	1			0.70
North East						1						0.10
North West												0.00
North Central Coast		1			1	1			1			0.40
South Central Coast					1	1						0.20
Central Highlands						1						0.10
South East	1	1			1	1			1			0.50
Mekong River Delta						1						0.10
Average	0.30	0.50	0.00	0.00	0.60	1.00	0.20	0.20	0.40	0.00	0.10	0.33

Note: Same as for Table 5.

Totals: See explanation in Table 7.

Source: Same as for Table 3.

Table 10

Bootstrap spatial FOD comparisons for Vietnam, 2006 (Probabilities).

	National	Rural	Urban	RRD	NE	NW	NCC	SCC	CH	SE	MRD	Avg.
National		1			0.46	1			0.02			0.25
Rural					0.27	0.99			0.01			0.13
Urban	1	1			0.98	1	0.72	0.48	0.98	0.01	0.51	0.67
Red River Delta	1	1			0.98	1	0.82	0.87	1	0.02	0.13	0.68
North East	0.00	0.00				0.59	0		0.01			0.06
North West												0.00
North Central Coast	0.08	0.41			0.73	1		0	0.3			0.25
South Central Coast	0.01	0.07			0.36	0.97	0		0.35			0.18
Central Highlands					0.03	0.92						0.10
South East	0.69	0.90			0.84	1	0.06	0.24	0.78		0.00	0.45
Mekong River Delta					0.05	0.87			0.02			0.09
Average	0.28	0.44	0.00	0.00	0.47	0.93	0.16	0.16	0.35	0.00	0.06	0.29

Note: Same as for Table 5.

Totals: See explanation in Table 7.

Source: Same as for Table 3.

Table 11

Spatial FOD comparisons for Mozambique, 2003.

	Nat	Rur	Urb	NI	CD	NA	ZA	TE	MA	SO	IN	GA	MP	MC	Avg.
National		1					1								0.15
Rural							1								0.08
Urban	1	1			1	1	1	1	1	1					0.62
Niassa															0.00
Cabo Delgado							1								0.08
Nampula															0.00
Zambezia															0.00
Tete							1								0.08
Manica		1					1								0.15
Sofala							1								0.08
Inhambane		1					1								0.15
Gaza		1			1		1								0.23
Maputo Province	1	1		1	1	1	1	1		1	1	1			0.77
Maputo City	1	1	1	1	1	1	1	1	1	1	1	1			0.92
Average	0.23	0.54	0.08	0.15	0.31	0.23	0.85	0.23	0.15	0.23	0.15	0.15	0.00	0.00	0.25

Note: Same as for Table 5.

Totals: See explanation in Table 7.

Source: Same as for Table 3.

Table 12

Bootstrap FOD comparisons for Mozambique, 2003 (Probabilities).

	Nat	Rur	Urb	NI	CD	NA	ZA	TE	MA	SO	IN	GA	MP	MC	Avg.
National		1				0.02	1	0.01							0.16
Rural							0.47								0.04
Urban	1	1		0.47	0.93	1	1	1.00	0.43	0.96	0.39	0.12			0.64
Niassa							0.18								0.01
Cabo Delgado		0.06		0.00			0.65	0.00							0.05
Nampula		0.04					0.48								0.04
Zambezia															0.00
Tete		0.19					0.87								0.08
Manica		0.95			0.02	0.01	0.99	0.11							0.16
Sofala		0.03				0.01	0.94								0.07
Inhambane	0.02	0.96		0.03	0.14	0.04	1.00	0.07							0.17
Gaza	0.03	0.97		0.05	0.37	0.16	1	0.20			0.10				0.22
Maputo Province	1	1	0.19	0.96	1	1	1	1	0.40	0.80	1	0.99			0.80
Maputo City	1	1	0.98	0.87	1	1	1	1	1	1	1.00	0.99	0.19		0.93
Average	0.23	0.55	0.09	0.18	0.27	0.25	0.81	0.26	0.14	0.21	0.19	0.16	0.01	0.00	0.26

Note: Same as for Table 5.

Totals: See explanation in Table 7.

Source: Same as for Table 3.

Table 13

Spatial FOD comparisons for Mozambique, 2008.

	Nat	Rur	Urb	NI	CD	NA	ZA	TE	MA	SO	IN	GA	MP	MC	Avg.
National		1													0.08
Rural															0.00
Urban	1	1		1	1	1	1	1							0.54
Niassa					1										0.08
Cabo Delgado															0.00
Nampula															0.00
Zambezia															0.00
Tete															0.00
Manica		1					1								0.15
Sofala															0.00
Inhambane															0.00
Gaza		1			1		1		1		1				0.38
Maputo Province	1	1		1	1	1	1	1	1		1				0.69
Maputo City	1	1	1	1	1	1	1	1	1	1	1	1			0.92
Average	0.23	0.46	0.08	0.23	0.38	0.23	0.38	0.23	0.23	0.08	0.23	0.08	0.00	0.00	0.22

Note: Same as for Table 5.

Totals: See explanation in Table 7.

Source: Same as for Table 3.

Table 14

Bootstrap FOD comparisons for Mozambique, 2008 (Probabilities).

	Nat	Rur	Urb	NI	CD	NA	ZA	TE	MA	SO	IN	GA	MP	MC	Avg.
National		1				0.09	0.07								0.09
Rural															0.00
Urban	1	1		0.72	0.94	1	1.00	0.93	0.33	0.20	0.02				0.55
Niassa					0.27		0.05								0.02
Cabo Delgado							0.00								0.00
Nampula		0.00													0.00
Zambezia															0.00
Tete															0.00
Manica		0.42				0.00	0.40	0.01							0.06
Sofala		0.04				0.00	0.13	0.01							0.01
Inhambane		0.30			0.04	0.00	0.31								0.05
Gaza	0.04	0.98		0.01	0.44	0.09	0.90	0.28	0.52	0.02	0.26				0.27
Maputo Province	0.99	1	0.00	0.38	0.95	1.00	1.00	0.97	0.65	0.07	0.59	0.03			0.59
Maputo City	1	1	1	1	1	1	1	1	1	1	1.00	0.55			0.89
Average	0.23	0.44	0.08	0.16	0.28	0.24	0.37	0.25	0.19	0.10	0.14	0.04	0.00	0.00	0.20

Note: Same as for Table 5.

Totals: See explanation in Table 7.

Source: Same as for Table 3.